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MATHEMATICAL MODELLING TASKS FOR HYDRAULIC RESISTANCE COEFFICIENT ESTIMATION USING NEURAL NETWORKS

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Abstract

This paper presents a comprehensive approach to estimating the Chézy roughness coefficient as a key parameter of hydraulic resistance in natural river channels. Based on the analysis of 43 well-known empirical and semi-empirical formulae for C, as well as 13 formulae for the Gauckler–Manning coefficient, the dependencies were systematised and classified into groups according to hydromorphological and hydraulic parameters.

An artificial neural network (ANN) was developed to estimate the coefficient C considering key hydromorphological factors. The model was validated using data from the Dnipro, Desna, and Prypiat rivers; the Nash–Sutcliffe Efficiency (NSE) values ranged from 0.94 to 0.98, with relative errors of 0.9–13.9%.

Additional testing was conducted on mountain rivers (Tysa, Teresva, Latorytsia, Opir, Rika, Chornyi Cheremosh), where anomalous values were excluded from the training datasets, improving prediction accuracy. It was demonstrated that the use of one- or two-layer ANNs is appropriate when high-quality training data are available.

To improve accuracy under limited data conditions, an ensemble model (ANN-A, ANN-B1, ANN-B2) was implemented using the bagging method. A strategy of independent training of networks was applied, followed by aggregation of outputs using majority voting. The testing results showed relative discharge errors ranging from 0.3% to 6.1%, and NSE values from 0.991 to 0.998.

The study confirms the high accuracy and practical applicability of the ensemble approach for estimating the Chézy coefficient in contexts with limited hydromorphological information.

Keywords: *Chézy coefficient, hydraulic resistance, artificial neural networks, model ensemble, mathematical modelling, river flows, hydromorphology, NSE, machine learning, hydraulic engineering structures.*

Hydraulic resistance refers to the forces with which the riverbed or channel bed opposes the motion of the water flow. The main factors influencing the magnitude of these forces include: roughness elements, bedforms and other obstacles, channel bends and curvature, variability in cross-sectional shape and size along the channel, suspended and bedload sediments, vegetation, ice phenomena, and others.

Typically, hydraulic resistance is evaluated using integral parameters such as roughness coefficients, Chézy coefficients, hydraulic friction coefficients, or roughness element height. The calculation of mean flow velocities in open channels, while accounting for hydraulic resistance, is one of the central problems in river hydraulics.

The aim of the study is to develop and experimentally validate an ensemble-based computational model using artificial neural networks (ANNs) for estimating the Chézy coefficient under various hydromorphological river conditions, in order to improve the reliability of hydraulic calculations and the robustness of engineering decision-making.

The mathematical modelling of the Chézy roughness coefficient for river flow conditions is considered using neural networks, encompassing a set of the following tasks.

1. A study of existing methods for estimating the Chézy roughness coefficient as a measure of hydraulic resistance in river channels [1]

As part of this study, we reviewed, analysed, and systematised a wide range of well-known and frequently cited empirical and semi-empirical formulae and relationships used to estimate the Chézy roughness coefficient. In total, 43 such formulae were examined, along with 13 additional formulae applicable to the estimation of the Gauckler–Manning roughness coefficient.

Based on these expressions, approximately 250 empirical equations can be constructed for determining the Chézy coefficient as a function of river and channel hydromorphological characteristics, hydraulic conditions, applicable ranges, and other factors.

The study resulted in the development of a general classification and systematisation of the main empirical and semi-empirical relationships, grouped as follows:

1. formulae primarily expressing the relationship between the Chézy coefficient C and the Manning roughness coefficient n, sometimes also incorporating the hydraulic radius or the average flow depth;
2. formulae in which hydraulic resistance is determined based on the height of channel roughness elements or the mean diameter of bed and bank material particles, or the height and length of bedforms;
3. formulae that account for the effect of water surface slope and average flow depth or hydraulic radius;
4. formulae expressing the dependence of the C coefficient on the average channel width and hydraulic radius (or mean flow depth).

Overall, it can be concluded that there is no ideal or universally applicable method for determining the Chézy roughness coefficient. Key challenges include uncertainties related to hydromorphological changes associated with sediment deposition and erosion processes in natural watercourses, as well as seasonal variations in aquatic and riparian vegetation, including floodplain dynamics.

Moreover, hydraulic resistance may vary due to the spatio-temporal variability of other hydraulic parameters. A critical factor in the reliable estimation of the Chézy coefficient using various empirical methods lies in the accuracy of field measurements of the input parameters used in the selected formulae, as these strongly influence the relative error of the final calculations.

2. Study and development of a data structure for training an artificial neural network in solving problems related to the calculation of the Chézy roughness coefficient [2]

As part of this task, the problem of structuring the data and developing general rules for the formation of training and testing datasets was addressed. These datasets are intended for training artificial neural networks (ANNs) developed to estimate the Chézy roughness coefficient, taking into account the parametric uncertainty of input data related to hydromorphological factors and parameters characterising hydraulic resistance in river channels.

The modelling was performed using a fully connected feedforward artificial neural network with a single hidden layer. The network architecture consisted of two input neurons, four neurons in the hidden layer, and one output neuron. A standard sigmoid transfer function was applied in the hidden layer neurons, while a linear activation function was used in the output neuron. Python was employed for network design, training, and testing.

It was proposed that both training and evaluation of the ANN for Chézy coefficient prediction should incorporate the following key hydromorphological parameters: the Gauckler–Manning roughness coefficient n and water surface slope S_f; average flow width B and depth h; height of bed roughness elements Δ and hydraulic radius R. It is assumed that multicollinearity among these parameters is either absent or negligible.

Considering the interdependencies between the Chézy coefficient (C) and the identified parameters, representative input parameters $(x_1,\ x_2)$and corresponding training datasets $(x_1,\ x_2, C)$ were prepared. Using these, the ANN computes the Chézy coefficient (C) as the dependent variable (1), where selected combinations of input parameters $x_1,\ x_2$ are treated as independent variables.

$$C = f(x_1,\ x_2), x_1 \in \{n, \Delta, S_f, B\}, x_2 \in \{h, R\}. \qquad (1)$$

The separation of parameters into two characteristic groups was carried out based on the analysis of various empirical formulae and relationships that can be used to determine the Chézy coefficient, drawing on the findings of the previous study [1].

Accordingly, the proposed computational model (1), based on a feedforward ANN with a sigmoidal logistic activation function, performs the approximation of continuous functions. The ANN is trained using the backpropagation algorithm.

The structuring of the network input data $(x_1,\ x_2)$ and training samples $(x_1,\ x_2, C)$ involved the construction of informative, smoothed, continuous, and normalised input data arrays, taking into account statistical uncertainty (e.g., errors, missing measurements, etc.).

Training and testing of the ANN were carried out using current hydromorphological data from selected lowland sections of the Dnipro, Desna, and Prypiat Rivers.

The adequacy of the proposed computational model for predicting the Chézy coefficient was demonstrated using the Nash–Sutcliffe Efficiency (NSE) coefficient, which is commonly applied in hydrological and hydraulic modelling and forecasting.

3. Study of the Effectiveness of Chézy Roughness Coefficient Estimation Using an Artificial Neural Network for Supporting Mathematical Modelling of River Flows [3]

The task of calculating the Chézy coefficient was addressed using the neural network proposed in the previous study [2], based on a limited set of field data on the hydrological and hydromorphological characteristics of selected sections of lowland rivers — the Dnipro, Desna, and Prypiat — as well as mountain rivers including the Tysa, Teresva, Latorytsia, Opir, Rika, and Chornyi Cheremosh.

It was proposed that, during the stage of domain analysis and field data collection for ANN training dataset construction, anomalous and incomplete data samples related to river reach characteristics should be excluded from consideration.

The procedure for evaluating the ANN's effectiveness was based on comparing observed Q_o and predicted Q_p water discharges (Q_p were determined using the Chézy coefficient C_p, estimated via the neural network).

Predicted values obtained using training samples containing anomalous data were associated with relative errors ranging from 3.8% to 18.1%.

The ANN trained on samples without anomalous data produced more accurate predictions, with relative errors ranging from 0.9% to 13.9%.

4. Study of the Optimal Architecture of a Multilayer Artificial Neural Network for Estimating the Chézy Roughness Coefficient [4]

In this task, the prediction of the Chézy roughness coefficient was performed in accordance with computational model (1), based on the neural network previously validated in studies [2, 3]. The investigation focused on fully connected feedforward artificial neural networks (ANNs) with one, two, and three hidden layers.

The training and testing of the different ANN configurations were carried out using datasets on the hydrological and hydromorphological characteristics of selected sections of both lowland and mountain rivers, developed within the framework of studies [2, 3].

Testing various ANN architectures produced the following key results. For calculating the Chézy coefficient within the proposed computational model, it is sufficient to use a feedforward ANN with one or two hidden layers and a sigmoidal logistic activation function, provided that a high-quality training dataset is available.

In the test cases based on data from [2, 3], the relative error in discharge predictions ranged from 0.9% to 13.9%, depending on the river. The Nash–Sutcliffe Efficiency (NSE) coefficient for model performance ranged from 0.94 to 0.98, indicating a level of accuracy sufficient for practical applications of Chézy coefficient estimation.

5. Ensemble Modelling of the Chézy Hydraulic Resistance Coefficient Using Artificial Neural Networks [5]

The task of estimating an approximate value of the Chézy hydraulic resistance coefficient $\widetilde{C}$ is considered using an ensemble of artificial neural networks (ANNs):

$$\tilde{C}(x_1, x_2) = C_A \pm \phi,\ x_1 \in \{n, \Delta, S_f, B\},\ x_2 \in \{h, R\}, \qquad (2)$$

where $C_A = f(x_1, x_2)$ – the value of the Chézy coefficient in the first approximation, calculated using the baseline ANN (ANN-A), which was validated in our previous studies [2–4]; $\phi = f(x_1, x_2)$ – a refinement term determined using two additional ANNs (ANN-B1 and ANN-B2), developed based on the baseline network model.

To solve problem (2), established methods from ensemble modelling theory are proposed. The ensemble approach is based on the idea of combining several base machine learning models into a unified structure to create a more robust composite model, which can outperform its individual components and significantly improve prediction accuracy.

In the context of problem (2), a homogeneous ensemble of neural network models—ANN-A, ANN-B1, and ANN-B2—was employed, along with a strategy of using independent datasets for each

ANN. The ensemble training and the creation of data subsets for the individual network models were carried out using the bagging (Bootstrap Aggregating) method. Accordingly, a parallel ensemble training technique was applied for the individual ANNs using the backpropagation algorithm.

The aggregation of output data (predictions) from the ensemble components into a single forecasted result, in accordance with computational model (2), was based on the majority voting method, with consideration of inverse problem solutions.

Python was used for the development, training, and testing of the neural network ensemble. The software implementation of the computational algorithms for training the ANN ensemble and predicting the Chézy roughness coefficient is presented in [6].

For the validation of the computational algorithm for hydraulic resistance coefficient prediction using the ANN ensemble, datasets on hydrological and hydromorphological characteristics of selected lowland and mountain river sections, as proposed in studies [2, 3], were utilised. The testing procedure consisted in comparing observed values Q_o and calculated (predicted) values Q_p based on the computed (predicted) values of the Chézy coefficient C_p, obtained using the artificial neural network ensemble.

It was shown that the relative errors of the predicted values *Q* ranged from 0.3% to 6.1%, depending on the river, and the values of the Nash–Sutcliffe Efficiency coefficient *NSE* were in the range of 0.991 to 0.998.

The validation results indicate that the computational accuracy is sufficient for practical applications and confirm the high predictive capability of the proposed ANN ensemble-based algorithm for estimating the Chézy coefficient.

Conclusions

The accuracy of hydraulic resistance parameter estimation is critically important for solving engineering problems related to the design of hydraulic structures and water resources management, regardless of the selected mathematical models or computational approaches.

Given the considerable variability of hydromorphological conditions in river systems, flow resistance varies over a wide range, directly affecting discharge capacity. In this context, the Chézy roughness coefficient C proves to be a more representative parameter of hydraulic resistance compared to other empirical characteristics, such as the Gauckler–Manning and Darcy–Weisbach coefficients. Its advantage lies in the ability to reflect the combined influence of morphological and hydrological factors.

However, no universal method for determining the Chézy coefficient C exists, and the accuracy of its estimation largely depends on the reliability (precision) of field measurements of relevant hydromorphological parameters in open-channel flows.

The results of Chézy coefficient modelling based on artificial neural networks have shown that, under conditions of limited input data, the application of ensemble approaches can reduce errors and improve the reliability of predictive estimates, helping to overcome the limitations inherent in individual models.

References

1. Stefanyshyn, D. V., Khodnevich, Y. V., Korbutiak, V. M. (2021). Estimating the Chezy roughness coefficient as a characteristic of hydraulic resistance to flow in river channels: a general overview, existing challenges, and ways of their overcoming. Екологічна безпека та природокористування, 39(3), 16–43. https://doi.org/10.32347/2411-4049.2021.3.16-43.

2. Yaroslav V. Khodnevych, Dmytro V. Stefanyshyn (2022). Data arrangements to train an artificial neural network within solving the tasks for calculating the Chezy roughness coefficient

under uncertainty of parameters determining the hydraulic resistance to flow in river channels. Екологічна безпека та природокористування, Том 42 № 2, 59-85. https://doi.org/10.32347/2411-4049.2022.2.59-85

3. Khodnevych, Y., Stefanyshyn, D., Korbutiak, V. (2023). The Chezy Roughness Coefficient Computing Using an Artificial Neural Network to Support the Mathematical Modelling of River Flows. In: Dovgyi, S., Trofymchuk, O., Ustimenko, V., Globa, L. (eds) Information and Communication Technologies and Sustainable Development. ICT&SD 2022. Lecture Notes in Networks and Systems, vol 809. Springer, Cham. https://doi.org/10.1007/978-3-031-46880-3_26

4. Yaroslav Khodnevych, Dmytro Stefanyshyn (2023). Do we need a more sophisticated multilayer artificial neural network to compute roughness coefficient? Екологічна безпека та природокористування, 48(4), 170–182. https://doi.org/10.32347/2411-4049.2023.4.170-182

5. Ходневич Я.В., Стефанишин Д.В. (2024). До питання обчислення коефіцієнта гідравлічного опору за допомогою ансамблю штучних нейронних мереж. Математичне моделювання та інформаційно-комунікаційні технології для зміцнення та відновлення // Колективна монографія за матеріалами 23 Міжнародної науково-практичної конференції «Інформаційно-комунікаційні технології та сталий розвиток» (Київ, 12-13 листопада 2024 р.) За заг. ред. С.О. Довгого. Київ, ТОВ «Видавництво «Юстон», 121-123. https://itgip.org/wp-content/uploads/2024/11/2024-11-24_zbirka_all_07_11_2024_148x210.pdf

6. Khodnevych, Ya. (2025). Software Implementation of a Computational Algorithm for Training an Ensemble of Neural Networks to Predict the Chezy Roughness Coefficient. Available from https://github.com/yakhodnevych/ANNE_approximation_C.git

ЗАДАЧІ МАТЕМАТИЧНОГО МОДЕЛЮВАННЯ КОЕФІЦІЄНТА ГІДРАВЛІЧНОГО ОПОРУ НА ОСНОВІ НЕЙРОННИХ МЕРЕЖ

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Анотація

У роботі представлено комплексний підхід до оцінки коефіцієнта шорсткості Шезі як ключової характеристики гідравлічного опору в природних річкових руслах. На основі аналізу 43 відомих емпіричних і напівемпіричних формул для *C*, а також 13 формул для коефіцієнта Гоклера–Меннінга, було здійснено систематизацію залежностей та класифікацію їх за групами залежно від гідроморфологічних та гідравлічних параметрів.Розроблено штучну нейронну мережу (ШНМ) для оцінки коефіцієнта *C* з урахуванням основних гідроморфологічних параметрів. Модель апробовано на даних для річок Дніпро, Десна та Прип'ять; коефіцієнт ефективності Неша–Сатткліффа (NSE) становив 0,94–0,98, а відносна похибка 0,9–13,9%.Додаткове тестування здійснено для гірських річок (Тиса, Тересва, Латориця, Опір, Ріка, Чорний Черемош) із виключенням аномальних значень з навчальних вибірок, що покращило точність прогнозів. Показано, що використання одно- або двошарових ШНМ є доцільним при якісному навчальному наборі.

Для підвищення точності в умовах обмеженого обсягу даних реалізовано ансамблеву модель (ANN-A, ANN-B1, ANN-B2) з методом беггінгу. Застосовано стратегію незалежного навчання мереж із подальшим об'єднанням результатів за принципом максимального голосування. За результатами тестування отримано відносні похибки витрати води в межах 0,3–6,1% та значення *NSE* від 0,991 до 0,998.

Результати дослідження підтверджують високу точність і практичну придатність ансамблевого підходу для оцінки коефіцієнта Шезі в умовах обмеженої гідроморфологічної інформації.

Ключові слова: коефіцієнт Шезі, гідравлічний опір, штучні нейронні мережі, ансамбль моделей, математичне моделювання, річкові течії, гідроморфологія, NSE, машинне навчання, гідротехнічні споруди.